**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

Z = (x-µ)/σ = (50-45)/8 = 0.625

P value = 0.7324(Probability of finishing the service)

So, the Probability that the service manager cannot meet his commitment = 1- (P value)

= 1 - 0.7324 = **0.2676**

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

This question can be answered in 2 ways.

(a) Z= (44-38)/6= 1

P value = 0.84134

Therefore, the Probability of employees older than 44 would 1-P Value = 0.15866(15.86%).

Probability of employees between the age of 38 and 44 = 0.5 - P(Employees>44) = 34.14%

Hence the above **statement is false.**

(b) As per the empirical rule, in a normal distribution, approx. 68% of the data lies within the range of µ±σ, which is 38±6. Hence approx. 34% of the employees are aged between 38 and 44, as the distribution will be almost symmetric. So, there will be less number of employees that are not older than 38+6(44). Hence the above **statement is false.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Z = (30-38)/6 = -1.33

P value = 0.09176 = 9.17%

Therefore, the expected number of employees under the age of 30 = 9.17% of 400 = 36.68≈36.

Hence the **statement is true**.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

X1 +X2 ~ N(µ1+µ2, σ12+σ22) ;

2X1 ~ N(2µ, 4σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

µ = 100, σ^2=20^2, σ=20

Probability between values a and b is 0.99. So the probability outside the values a and b = 1-0.99 =0.01

Since a and b are symmetric about the mean, the probability of the area below a is -0.005 and the probability of the area above b is +0.005

From the Z-table we can find the value of Z for the P-Value of 0.005. It is ±2.57.

Z = (X-µ)/σ and X = (Z\*σ) + µ

a = (-2.57\*20) +100 = 48.6

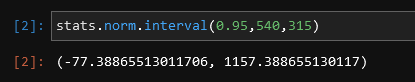
b = (2.57\*20) +100 = 151.4

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
8. Specify the 5th percentile of profit (in Rupees) for the company
9. Which of the two divisions has a larger probability of making a loss in a given year?

µ = 5+7 = $12 Million = 12 \* 45 = **₹540 Million is the Mean Profit**

σ = 3+4 = $7 Million = 7 \* 45 = **₹315 Million is the Standard Deviation**

A.



B.

(5 percentile is 0.995/2 = 0.4975. Therefore Z = -1.64)

Z = (X-µ)/σ

X = σ.Z + µ = 315 \* (-1.64) + 540 = **₹23.4 Million**

C.

